3.8 Order Statistics

Suppose the times of arrival (arranged in order) are $T_1, T_2 \ldots T_N$. Working out the probability of getting this set of data takes a little care; the chance of getting a datum less than t_0 is of course zero. For brevity, introduce the Heaviside function

$$\begin{aligned} H(x) &= 1 \text{ x} > 0 && (1) \\ &= 0 \text{ x} \leq 0 && (2) \end{aligned}$$

The probability of getting our set of data is proportional to

$$\mathcal{L} = \prod_i H(T_i - t_0) \exp -(T_i - t_0).$$

Viewed as a function of t_0, this product is zero for $t_0 > T_1$ and has its maximum at T_1. Note that $\mathcal{L}$ is not actually differentiable at T_1.
So an estimate of t_0 is just T_1, the time of arrival of the first neutrino. This, despite appearances, does use all of the data because we have arranged $T_1 < T_2 < T_3 \ldots$
Now use Eqn 3.17 to get the distribution of the minimum (n=1 in that formula). The answer is simple:

$$\text{prob}(t_1) = N \exp -N(t_1 - t_0)$$

and the expected value

$$\int t_1 \, \text{prob}(t_1) \, dt_1 = t_0 + \frac{1}{N}$$

which means that this estimate of t_0 is biassed, since it gives the wrong answer on average, but consistent, because it gets closer and closer to the right answer as we get more data.